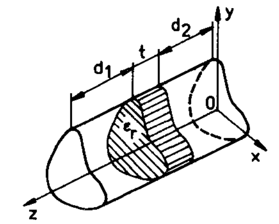

Fig 1 Configuration discussed in this paper.

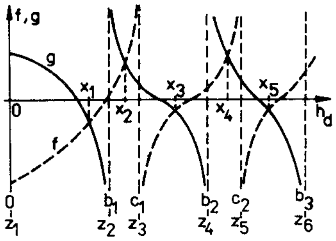

Fig. 2. Graphical solution of (7). The elements c_i, b_j, z_k are shown in the lower part of the diagram.

It can be proved (Appendix II) that for j we must take

$$j = s - i - 1 \tag{10}$$

where s is the sequence number of the sought root and

$$i = \begin{cases} \text{int}[h_a(d_1+d_2)/\pi] & h_a^2 \geqslant 0, \quad \text{TE modes} \quad (11a) \\ 1+\text{int}[h_a(d_1+d_2)/\pi] & h_a^2 \geqslant 0, \quad \text{TM modes} \quad (11b) \\ 0 & h_a^2 < 0. \quad (11c) \end{cases}$$

(int x is the greatest integer value of x). The bounds of the required interval are

$$u_s = j\pi/t, \quad v_s = (j+1)\pi/t. \tag{12}$$

III. Conclusion

The presented method of defining one-root intervals quickly and reliably determines the roots of dispersion relations for any chosen mode of resonance without ambiguity and represents an effective tool for systematic computer analysis of the structure.

Appendix I
Dispersion Relations

In order for the transverse field components to be continuous at the air–dielectric interfaces of the resonator, certain specific relation between the wavenumbers h_a and h_d must be satisfied. This relation is referred to as a dispersion relation. For the TE modes of the discussed resonator, the dispersion relation is

$$\frac{\sin h_d t}{h_d}\left[\left(\frac{h_d}{h_a}\right)^2 \sin h_a d_1 \sin h_a d_2 - \cos h_a d_1 \cos h_a d_2\right] - \cos h_d t \frac{\sin h_a(d_1+d_2)}{h_a} = 0 \tag{A1}$$

and for the TM modes, it is

$$\frac{\sin h_d t}{h_d}\left(h_d^2 \cos h_a d_1 \cos h_a d_2 - \epsilon_r^2 h_a^2 \sin h_a d_1 \sin h_a d_2\right) + \epsilon_r h_a \cos h_d t \sin h_a(d_1+d_2) = 0. \tag{A2}$$

Appendix II
Proof of Formulas (10) and (11)

The lower bound $u_s = z_s$ of a one-root interval for root x_s is identical with the sth member of the sequence $\{z_k\}$, whose members z_1 through z_s can be obtained by arranging the elements

$$0, c_1, c_2, \cdots, c_i, b_1, b_2, \cdots, b_j \tag{A3}$$

into a nondecreasing (finite) sequence. The overall number of elements used in (A3) is $s = i + j + 1$, hence (10) is valid. Last used poles being b_j and c_i implies both (9) and $c_i \leqslant x_s < c_{i+1}$, which is for $h_a \geqslant a_1$ equivalent with $a_i \leqslant h_a < a_{i+1}$ and results in (11a,b) after simple calculations. If $h_a < a_1$ ($a_1 \neq 0$ for TE modes only) or if $h_a^2 < 0$, no element from the set $\{c_i\}$ is used in (A3), so that $i = 0$. The result $i = 0$ is obtained from (11a) for $h_a < a_1$ and from (11c) for $h_a^2 < 0$.

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Normal-Mode Parameters of Microstrip Coupled Lines of Unequal Width

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Abstract —**Empirical relations for the capacitive and inductive coupling coefficients have been used to compute normal-mode parameters of nonidentical microstrip coupled lines. The computed values of mode velocities and mode impedances are compared with available results from the literature and experimental results on nonidentical microstrip couplers to test the applicability of the empirical relations.**

I. Introduction

The normal-mode parameters of microstrip coupled lines are usually determined from the capacitances and inductances of the microstrip lines [1]–[6]. These are found by solving Laplace's equation for the quasi-TEM case and the Helmholtz equation for the dispersive case [1]. Tripathi and Chang [2] calculated self and mutual capacitances for nonidentical microstrip coupled lines using Green's function integral equation method. This short paper aims at providing simple relations for finding the normal-mode parameters of nonidentical lines.

A previous communication [7] described the use of the empirical relations for inductive and capacitive coupling coefficients of identical coupled microstrip lines. These have now been modified to enable calculation of the normal-mode parameters of nonidentical microstrip coupled lines from a knowledge of the dimensional ratio of the lines and relative dielectric constant of the substrate material. Values of normal-mode parameters thus computed from the quasi-static approach have been compared with

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those obtained by Tripathi and Chang [2]. Nonidentical microstrip couplers have been realized on alumina substrates and the experimental results compared with the theoretical values.

II. Numerical Computation of Normal-Mode Parameters

The capacitive and inductive coupling coefficients of identical microstrip coupled lines [7] have been modified for the nonidentical case and may be written as

$$k_c = 0.55\exp[-(A_1 S/H + B_1(W_1+W_2)/2H)] \quad (1)$$

$$k_L = 0.55\exp[-(A_2 S/H + B_2(W_1+W_2)/2H)] \quad (2)$$

where

$$A_1(\epsilon_r) = 1 + \frac{1}{4}\ln\left(\frac{\epsilon_r+1}{2}\right) \quad B_1(\epsilon_r) = \frac{1}{10}\sqrt{\epsilon_r+1}$$

$$A_2(\mu_r) = 1 + \frac{1}{4}\ln\left(\frac{\mu_r+1}{2}\right) \quad B_2(\mu_r) = \frac{1}{10}\sqrt{\mu_r+1} \quad (3)$$

and where

- W_1, W_2 width of the two microstrip lines,
- S gap spacing between the two lines,
- H substrate thickness,
- μ_r, ϵ_r relative permeability and relative dielectric constant of the material of the substrate. For dielectric substrate, μ_r is unity, but for magnetic substrate, μ_r changes with magnetic field.

The conventional even- and odd-mode concepts cannot be directly used in case of nonidentical coupled lines. These modes can be defined only for special cases where the line parameters satisfy certain restrictive relationships [8]–[10]. The solutions of the eigenvalue problem of nonidentical coupled lines lead to four roots of the propagation constant and are characterized by the C- and π-modes [8]. The expressions for the ratio of wave voltages and mode impedances of the two lines for the C- and π-modes have been derived following Krage and Haddad [11]. These are

$$R_{c,\pi} = \sqrt{\frac{Z_{02}}{Z_{01}}}\,\frac{\sqrt{\beta_1\beta_2}(\beta_1 k_L - \beta_2 k_c)}{\beta_{c,\pi}^2 - (\beta_2^2 - k_L k_c \beta_1\beta_2)} \quad (4)$$

$$Z_{c,\pi}(1) = Z_{01}\frac{\beta_{c,\pi}(\beta_2 k_L - \beta_1 k_c)}{k_L\beta_1\beta_2(1-k_c^2) - \beta_{c,\pi}^2 k_c} \quad (5)$$

$$Z_{c,\pi}(2) = Z_{02}\frac{\beta_{c,\pi}(\beta_1 k_L - \beta_2 k_c)}{k_L\beta_1\beta_2(1-k_c^2) - \beta_{c,\pi}^2 k_c}. \quad (6)$$

Z_{01} and Z_{02} are uncoupled impedances of line 1 and line 2, respectively, while β_1 and β_2 are phase constants of the two lines. The proximity effect on Z_{01}, Z_{02}, β_1, and β_2 has been neglected. The phase velocities of the C- and π-modes may be obtained from the values of β_c and β_π, which are given by

$$\beta_{c,\pi} = \beta_0\sqrt{1 \pm \delta} \quad (7)$$

where

$$\beta_0^2 = \left(\frac{\beta_1^2+\beta_2^2}{2} - k_L k_c \beta_1\beta_2\right)$$

and

$$\delta^2 = \left(1 - \frac{\beta_1^2\beta_2^2(1-k_c^2)(1-k_L^2)}{\beta_0^4}\right).$$

Using the relations (4)–(7), the normal-mode parameters of nonidentical coupled lines have been computed for typical values of the linewidth ratio of the two lines, gap spacings, and the relative dielectric constant of the substrate material. Fig. 1 depicts the variation of Z_{c1}, $Z_{\pi 1}$, Z_{c2}, $Z_{\pi 2}$, V_c, and V_π with the ratio of auxiliary linewidth to main linewidth (W_2/W_1) for different S/H values. It shows that the impedances and phase velocities for the two modes approach each other for a weak coupling when the spacing between the two lines increases. The change in mode velocities with asymmetry is not very significant. Fig. 2 shows the variations in normal-mode parameters as a function of the dielec-

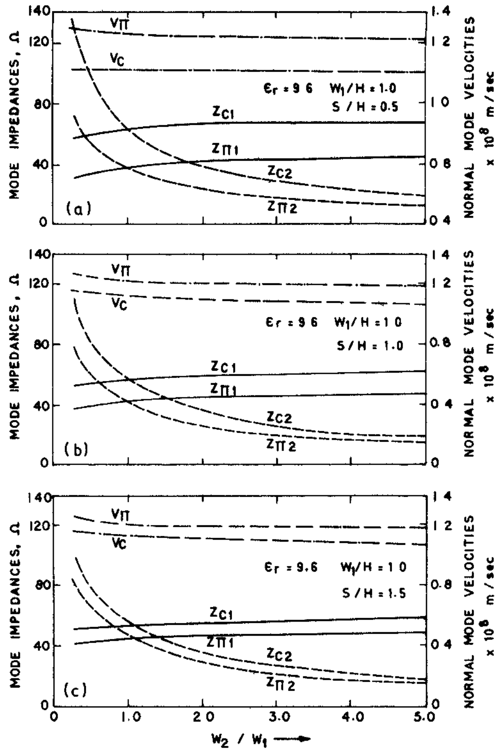

Fig. 1. Normal-mode parameters as a function of microstrip widths.

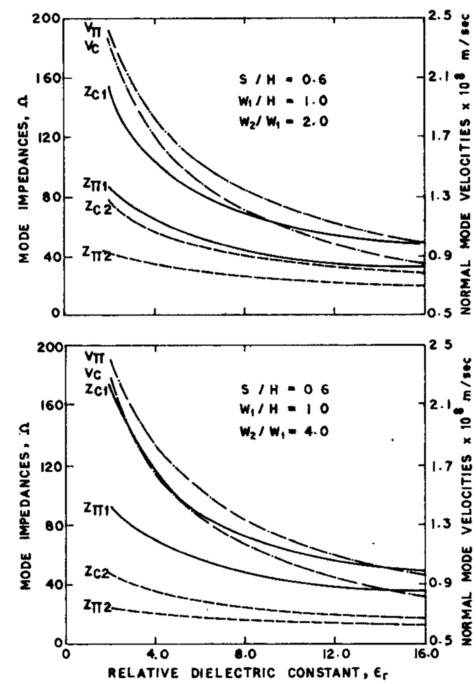

Fig. 2. Variation in normal-mode parameters as a function of dielectric constant.

TABLE I
COMPARISON OF COUPLED LINE PARAMETERS OF NONIDENTICAL LINE MICROSTRIP COUPLER

ϵ_r	Dimension $W_1/H = 1.0$		TRIPATHI and CHANG						This Method					
	S/H	W_2/W_1	Z_{c1} Ω	$Z_{\pi 1}$ Ω	Z_{c2} Ω	$Z_{\pi 2}$ Ω	$V_c \times 10^8$ m/sec	$V_\pi \times 10^8$ m/sec	Z_{c1} Ω	$Z_{\pi 1}$ Ω	Z_{c2} Ω	$Z_{\pi 2}$ Ω	$V_c \times 10^8$ m/sec	$V_c \times 10^8$ m/sec
6.0	0.4	3.0	87	50	37.68	21.66	1.39	1.55	91.6	52.68	36.33	20.89	1.386	1.54
10.0	0.2	3.0	71	38	28.9	15.46	1.07	1.24	72.93	39.19	29.63	15.92	1.079	1.252
	0.6	2.0	60	41	36.95	25.25	1.08	1.22	62.58	41.41	37.33	24.7	1.087	1.216
	0.6	2.6	61.5	42.5	30.39	21.83	1.07	1.21	63.45	42.38	30.77	20.56	1.073	1.209
	0.6	3.0	62.5	43.5	26.94	18.75	1.065	1.21	63.76	42.88	27.59	18.56	1.063	1.206
	1.0	2.0	56.5	43.0	34.8	26.48	1.075	1.22	57.5	44.1	35.36	27.12	1.085	1.19
	1.0	3.0	58.0	46.0	26.4	20.93	1.07	1.20	58.9	45.2	26.43	20.27	1.066	1.18
14.0	0.4	3.0	55.0	35.0	24.45	15.56	0.88	1.05	55.5	35.06	24.13	15.23	0.898	1.051
16.0	0.4	3.0	50.0	32.0	22.23	14.22	0.80	1.00	51.44	32.87	22.59	14.43	0.837	0.988

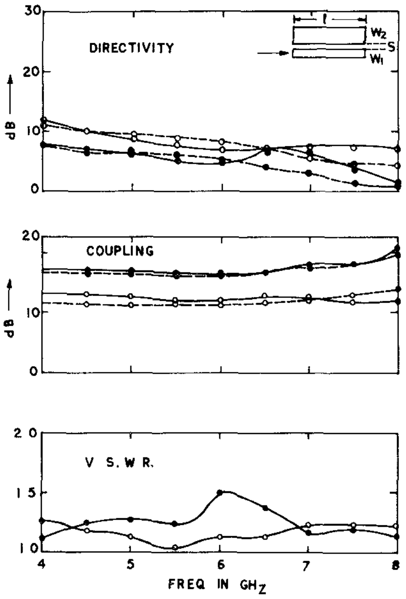

Fig. 3. Experimental and theoretical performance of two nonidentical line microstrip couplers. Coupler A —○— $W_1/H = 0.945$, $W_2/H = 1.89$, $S/H = 0.315$. Coupler B —●— $W_1/H = 0.945$, $W_2/H = 1.89$, $S/H = 0.819$. Experiment ——. Theory ----.

tric constant of the substrate (ϵ_r) for a given asymmetry. As expected, there is a significant variation in mode velocities with ϵ_r. The computed values of mode impedances and normal-mode velocities for different S/H, W_2/W_1, and ϵ_r values are compared in Table I with those of Tripathi and Chang [2]. The values of the parameters obtained from the two methods are quite close (within 5 percent). However, for the case of tight coupling ($S/H < 0.2$), the agreement is rather poor.

III. EXPERIMENTAL RESULTS AND DISCUSSION

Two nonidentical couplers of 11- and 16-dB coupling at the center frequency of 5.5 GHz have been designed on 25-mil-thick (H) alumina substrate ($\epsilon_r = 9.6$). The length of the coupled region L has been chosen such that at the center frequency $(\beta_c + \beta_\pi)l/2 = \pi/2$. At 5.5 GHz, l turns out to be 0.544 cm. The ratio of auxiliary linewidth to main linewidth for both the couplers is 2, and the input is applied to the main line of smaller width ($W/H = 0.945$). The performance of the couplers was measured in the frequency range 4–8 GHz using an HP 8755 coax reflectometer system, and the results are given in Fig. 3 (solid line). Theoretical calculations of coupling and directivity using the relations (1)–(7) in the same frequency range are shown in the same figure (dotted line). A comparison of experimental and theoretical results of coupling and directivity of the two nonidentical line couplers shows a close agreement around the center frequency.

IV. CONCLUSIONS

A method of computing normal-mode parameters using the empirical relations of coupling coefficients of nonidentical coupled lines has been presented. The computed values have been compared with Tripathi and Chang. The agreement between theoretical and experimental performance of two couplers made on alumina substrate is found to be good. The principal limitations of the method are that this is applicable for loose coupling ($S/H < 0.2$) and for substrates having values of relative dielectric constant greater than 6.

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An Analysis of a Width-Modulated Microstrip Periodic Structure

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***Abstract* —The wave propagation along a microstripline with sinusoidally varying width has been investigated. The analysis employs the circuit theory of uniform microstriplines and their step junctions. The wave**

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